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**FLAMES IN DUSTY MIXTURES -- THEIR  
STRUCTURE AND STABILITY**

**J. Buckmaster  
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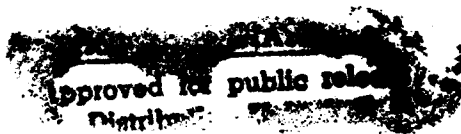
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# FLAMES IN DUSTY MIXTURES – THEIR STRUCTURE AND STABILITY

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## ABSTRACT

The structure and stability of flames in dusty mixtures is investigated. The presence of the dust leads to significant transport of energy by radiation and the fundamental goal of the analysis is to explore to what extent this displaces the classical non-hydrodynamical stability boundaries of the plane deflagration. An approximate description of the radiative transport permits analysis for arbitrary values of both the Planck length and the Boltzman number. It is shown that the pulsating/traveling-wave instability usually associated with values of Lewis number ( $Le$ ) bigger than 1 is strongly enhanced by the presence of radiation and can be present even if  $Le < 1$ . On the other hand radiation tends to suppress the cellular instability normally associated with values of  $Le$  less than 1. The latter is consistent with preliminary experimental observations of Abbud-Madrid and Ronney.

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## Introduction

We are concerned with plane flames supported by mixtures that contain a large number of small inert particles (dusty gases). The particles substantially influence the radiative transport and make emission and absorption energetically significant. Partial motivation for the study comes from recent experimental observations which suggest that in combustion fields of this kind cellular instability (the Turing instability associated with values of Lewis number less than 1, [1]) can be suppressed [2]. A discussion of stability and the effect of particle loading on the location of the stability boundaries in the wave-number - Lewis-number plane is the central contribution of this paper.

The presence of particles and radiative transport presents serious technical obstacles to an analytical treatment, and these must be overcome by judicious modeling. We start by assuming that the mass loading is small and the only impact of the particles is on the radiative transport. The bulk particle density is  $(4/3) \pi d^3 N \rho$  ( $N$  is the number density,  $d$  the particle diameter,  $\rho$  the substantive particle density) and the radiation effect is controlled by  $Nd^2$  so that our treatment is formally valid in the limit  $N \rightarrow \infty, d \rightarrow 0, Nd^2$  fixed. (Then  $Nd^3 \rightarrow 0$ )

A strategy for dealing with radiative transport which has been usefully employed by Joulin and his colleagues in a number of important studies (e.g. [3], [4]) is to assume that the Boltzman number is small, but we wish to avoid the restriction here since it is of limited applicability. Instead we start with the differential - equation approximation (the Eddington approximation)

$$L \nabla (\underline{L} \nabla \cdot \underline{q}) = \underline{3j} + 4 L \nabla T^4. \quad (1)$$

Since the nonlinearities here present serious difficulties we approximate the Planck length  $L$  by a constant and linearize the emission term, replacing Eqn. (1) by

$$L^2 \nabla (\underline{L} \nabla \cdot \underline{q}) = \underline{3j} + 16 T_c^3 L \nabla T \quad (2)$$

where  $T_c$  is a constant characteristic (emitting) temperature. Then  $\underline{q}$  is irrotational so that we may introduce a generating function  $\psi$  so that

$$\underline{q} = \underline{\nabla} \psi. \quad (3)$$

Equation (2), after a single integration, is then equivalent to

$$L^2 \nabla^2 \psi = \underline{3j} + 16 T_c^3 L (T - T_f) \quad (4)$$

where  $T_f$  is the remote cold-mixture temperature.

The other ingredients of our model are familiar and of proven merit. They are embodied in the equations

$$\begin{aligned} \rho (Y_t + u Y_x) &= \rho D \nabla^2 Y - B e^{-E/2R} \delta(n) \\ \rho C_p (T_t + u T_x) &= \lambda \nabla^2 T - \nabla^2 \psi + Q B e^{-E/2R} \delta(n) \end{aligned} \quad (5)$$

Here  $Y$  is the mixture mass fraction,  $T$  the temperature, and  $u$  is the steady flame speed so that the unperturbed flame is fixed. Reaction is modeled by a delta-function whose strength is an Arrhenius

function of the flame-temperature  $T_*$ ;  $n$  is the distance measured along the flame-sheet normal that points towards the burnt gas. And for the purposes of the stability analysis we adopt the constant-density model so that hydrodynamic effects are discarded.

Boundary conditions are:

$$\underline{x \rightarrow -\infty} \quad T \rightarrow T_f, \quad Y \rightarrow Y_f, \quad \psi \rightarrow 0 \quad (6)$$

We wish to construct the steady planar solution of the system (4) - (6) and to examine the linear stability of this solution to arbitrary perturbations. To this end it is convenient to introduce a flame-fixed coordinate system by replacing  $x$  by  $s$  where

$$s = x - Fy, \quad (7)$$

and the flame-sheet is located at  $s = 0$ . Equations (5) for  $s \neq 0$  are then

$$\rho [Y_t + (u - F_t) Y_s] = \rho D \nabla^2 Y, \quad (8)$$

$$\rho C_p [T_t + (u - F_t) T_s] = \lambda \nabla^2 T - \nabla^2 \psi$$

where

$$\nabla^2 = (1 + F_y^2) \partial^2 / \partial s^2 + \partial^2 / \partial y^2 - 2F_y \partial^2 / \partial y \partial s - F_{yy} \partial / \partial s \quad (9)$$

The jump conditions at the flame-sheet are:

$$[\psi] = 0, \quad [\psi_s] = 0$$

$$[T] = 0, [Y] = 0 \quad (10)$$

$$(1 + F_y^2)[T_s] = -(QBA) \exp(-E_2 RT_s), (1 + F_y^2)[Y_s] = (B \rho D) \exp(-E_2 RT_s).$$

### Steady Solution

The steady solution is constructed by seeking solutions for which

$$F = 0, T = T^s(s), Y = Y^s(s), \psi = \psi^s(s) \quad (11)$$

Consider the cubic

$$\alpha(\omega L)^3 - (\omega L)^2 - (3\alpha + \beta)(\omega L) + 3 = 0 \quad (12)$$

$$\text{where } \alpha = \lambda / L M C_p, \beta = 16 T_c^4 / M C_p T_c. \quad (13)$$

$\alpha$  is the familiar length  $\lambda / M C_p$  divided by the Planck length;  $\beta$  is a Boltzman number.

Equation (12) only has real roots, two of which are positive ( $\omega_1, \omega_2$ ), one negative ( $\omega_3$ ). In addition we define  $\omega_4$  by

$$\omega_4 = \rho u / \rho D = M / \rho D. \quad (14)$$

Then, without using the connection conditions (10), the steady solution has the form:

$$\begin{aligned} s < 0 \quad T^s &= T_f + C_1 e^{\omega_1 s} + C_2 e^{\omega_2 s} \\ \psi_s &= C_1 (M C_p - \lambda \omega_1) e^{\omega_1 s} - C_2 (M C_p - \lambda \omega_2) e^{\omega_2 s} \end{aligned} \quad (15)$$



$$Y^s = Y_f + C_4 e^{\omega_4 s}$$

$$s > 0 \quad T^s = T_a + C_3 e^{\omega_3 s}$$

$$\psi_s = C_3 (M C_p - \lambda \omega_3) e^{\omega_3 s} \quad (16)$$

$$Y = 0 .$$

Here  $T_a \equiv T_f + Q_f / C_p$  is the adiabatic flame temperature, the temperature at  $s \rightarrow \infty$  fixed by global energy conservation.

The various constants  $\{C_i\}$  are determined by use of the connection conditions (10 a-d) whence

$$C_4 = Y_f \quad (17)$$

and, with the definitions

$$z_i = \alpha L \omega_i, \quad D_i = C_i (T_a - T_f) \quad (18)$$

we have

$$D_1 + D_2 - D_3 = 1$$

$$D_1 z_1 + D_2 z_2 - D_3 z_3 = 1 \quad (19)$$

$$D_1 z_1^2 + D_2 z_2^2 - D_3 z_3^2 = 1$$

Equations (19) have solution

$$D_1 = (z_2 - \beta)(z_3 - \beta)(z_2 - z_1)^{-1}(z_3 - z_1)^{-1}$$

$$D_2 = (z_1 - \beta)(z_3 - \beta)(z_1 - z_2)^{-1}(z_3 - z_2)^{-1}$$

$$D_3 = -(z_1 - \beta)(z_2 - \beta)(z_1 - z_3)^{-1}(z_2 - z_3)^{-1} \quad (20)$$

In order to determine the mass flux  $M$  (i.e. the flame speed) it is convenient to define  $M_0$  by

$$M_0 = M(\beta = 0) \quad (21)$$

and since  $T_s = T_a$  at this limit, Eqn. (10e) is

$$[T_s] = (M_0 C_p / \lambda) (T_a - T_f) \exp[E / 2RT_a - E / 2R] \quad (22)$$

It follows that

$$M / M_0 = \exp\{(E/2RT_a) D_3 [D_3 + T_a / (T_a - T_f)]^{-1}\}. \quad (23)$$

$D_3$  is defined by the roots  $z_1, z_2$  and  $z_3$  which, in turn, are defined by  $\alpha$  and  $\beta$ , both of which depend on  $M$ . Thus Eqn. (23) is an implicit formula for the flame-speed.

*The limit  $\alpha \rightarrow \infty$*

It is of interest to examine the realistic limit  $\alpha \rightarrow \infty$ , when the Planck length is much larger than the diffusive scale (i.e. the Reynolds number based on  $L$  is large). Then the temperature distribution is

$$\begin{aligned} \underline{s < 0} \quad T^s &= T_f + (T_a - T_f) \left[ \exp \left( (MC_p / \lambda) s \right) + f_+(\beta, s) \right], \\ \underline{s > 0} \quad T^s &= T_a + (T_a - T_f) f_-(\beta, s). \end{aligned} \quad (24)$$

where

$$f_{\pm}(\beta, s) = \beta (\beta^2 + 12)^{-1/2} \exp \left\{ \left[ \left( -\frac{1}{2} \beta \pm \frac{1}{2} \sqrt{\beta^2 + 12} \right) / L \right] s \right\}.$$

On the scale  $s = O(L)$  there is a radiative preheat and *postheat* zone separated by a thin region ( $s = O(\lambda/MC_p)$ ) in which the temperature increases from

$$T_1 \equiv T_f + (T_a - T_f) \beta (\beta^2 + 12)^{-1}. \quad (25)$$

to

$$T_* \equiv T_a + (T_a - T_f) \beta (\beta^2 + 12)^{-1/2} \quad (26)$$

The flame-speed is the adiabatic flame speed corresponding to a cold mixture of temperature  $T_1$  so that

$$M / M_o = \exp \left\{ (E/2RT_a) \beta \left[ \beta + T_a (T_a - T_f)^{-1} (\beta^2 + 12)^{1/2} \right] \right\}. \quad (27)$$

In order to discuss this formula it is convenient to introduce the *virtual Boltzman number*

$$\beta_o = 16 T_c^4 / M_o C_p T_c. \quad (28)$$

Using  $\text{CH}_4/\text{air}$  adiabatic flame calculations of Giovangigli and Smooke [6] and with  $T_c$  equal to the adiabatic flame temperature we can plot variations of  $\beta_0$  with equivalence ratio, and these are shown in Fig. 1. This defines a realistic range of values of  $\beta_0$ .

Figure 2 shows variations of  $M/M_0$  and  $\beta$  with  $\beta_0$  determined from Eqns. (27), (28). The enhancement in the burning rate was described in ref. [3] in the case of small  $\beta$  ( $\beta = O(RT_a/E)$ ) and we call it the *Joulin effect*. It is analogous in its physical origins to the enhancement described by Weinberg in so-called excess enthalpy flames [5]. Here the temperature at the reaction zone is raised above the adiabatic flame temperature by radiative preheating, whereas in Weinberg's flames the increment is generated by providing a heat conduction path superior to that of the gas.

*The limit  $\alpha \rightarrow 0, \beta \rightarrow \infty, \alpha\beta = O(1)$*

The formulas (24) and (27) are not uniformly valid in  $\beta$ . Indeed, there is a distinguished limit  $\alpha \rightarrow 0, \beta \rightarrow \infty, \alpha\beta = O(1)$ . In this limit

$$\begin{aligned}\omega_1 L &\sim (2\alpha)^{-1} [1 + \sqrt{1 + 4\beta}] , \quad \omega_2 L \sim \alpha / \alpha\beta \\ \omega_3 L &\sim (2\alpha)^{-1} [1 - \sqrt{1 + 4\beta}]\end{aligned}\tag{29}$$

Thus the radiative preheat zone has thickness  $O(L^2 MC_p / \lambda)$  whereas the postheat zone has thickness  $O(\lambda / MC_p)$ . The temperature rises slowly from  $T_f$  to  $T_a$ ; increases rapidly from  $T_a$  to

$$T. \sim T_a + (T_a - T_f)(1 + 4\beta)^{-1/2} \quad (30)$$

and then decreases rapidly to  $T_a$ . The burning rate is given by the formula

$$M / M_o = \exp\left\{(E/2RT_a)\left[1 + T_a(T_a - T_f)^{-1}\sqrt{1 + 4\beta}\right]^{-1}\right\} \quad (31)$$

It is well known that in the limit of infinite activation energy the reaction zone is unstable if the temperature falls off towards the burnt gas on the same scale ( $O(\lambda/MC_p)$ ) as it falls off towards the fresh gas [7]. This is why, for example, the so-called premixed-flame domain of diffusion flame burning, for which the burning rate is a decreasing function of Damkohle number (a portion of the middle branch of the familiar S-shaped response), is unstable. Thus the steady solution described in this section may not be physically realizable.\*

#### *$\alpha$ Not Small*

When  $\alpha$  is not small we return to Eqn. (23) in order to calculate the burning rate. Some curves are shown in Fig. 2. It is noteworthy that for high particle loadings, when  $\alpha$  is not small, and for large values of the virtual Boltzman number, there is little augmentation of the burning velocity. This result should be compared with the

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\* The reaction zone instability occurs on a fast-time scale with the combustion field beyond the reaction zone unchanged. It is not captured by the stability calculations of the present paper.

saturation observed experimentally [2]. Note that in an experiment  $\beta_0$  is varied by changing the equivalence ratio, and  $\alpha_0$  can be varied with  $\beta_0$  fixed by changing  $L$  (i.e. by changing the particle loading).

### Stability Analysis

We seek infinitesimal perturbations of the steady solution by writing

$$\begin{aligned} T &= T^s + \varepsilon T_0(s) e^{iky + \omega t} \\ Y &= Y^s + \varepsilon Y_0(s) e^{iky + \omega t} \\ \psi &= \psi^s + \varepsilon \psi_0(s) e^{iky + \omega t} \\ F &= \varepsilon \delta e^{iky + \omega t} \end{aligned} \quad (32)$$

where  $\varepsilon \rightarrow 0$ .

The coefficient functions satisfy the equations:

$$\begin{aligned} \rho C_p [\omega T_0 + u T_0' - \delta \omega T^s] &= \lambda [T_0'' - k^2 T_0 + \delta k^2 T^s] \\ &\quad - [\psi_0'' - k^2 \psi_0 + \delta k^2 \psi^s] \\ \rho [\omega Y_0 + u Y_0' - \delta \omega Y^s] &= \rho D [Y_0'' - k^2 Y_0 + \delta k^2 Y^s] \\ L^2 [\psi_0'' - k^2 \psi_0 + \delta k^2 \psi^s] &= 3\psi_0 + 10 T_c^3 L T_0 \end{aligned} \quad (33)$$

Solutions are:

$$\underline{s < 0} \quad T_0 = A_1 e^{\gamma_1 s} + A_2 e^{\gamma_2 s} + \delta G_1 e^{\omega_1 s} + \delta G_2 e^{\omega_2 s}$$

$$Y_0 = C e^{\mu s} + \delta G_7 e^{\omega_4 s} \quad (34)$$

$$\psi_0 = B_1 e^{\gamma_1 s} + B_2 e^{\gamma_2 s} + \delta G_4 e^{\omega_1 s} + \delta G_5 e^{\omega_2 s}$$

$$s > 0 \quad T_0 = A_3 e^{\gamma_3 s} + A_4 e^{\gamma_4 s} + \delta G_3 e^{\omega_3 s}$$

$$Y_0 = 0, \quad (35)$$

$$\psi_0 = B_3 e^{\gamma_3 s} + B_4 e^{\gamma_4 s} + \delta G_6 e^{\omega_3 s}$$

Here  $\gamma_1, \gamma_2, \gamma_3$  and  $\gamma_4$  are the roots of the quartic

$$(\gamma L)^4 - \alpha^{-1} (\gamma L)^3 - (\gamma L)^2 [2(kL)^2 + 3 + \alpha^{-1} (\beta + \omega L u^{-1})] + (\gamma L) \alpha^{-1} [(kL)^2 + 3]$$

$$+ (kL)^4 + (kL)^2 (3 + \alpha^{-1} (\beta + \omega L u^{-1})) + 3 \alpha^{-1} \omega L u^{-1} = 0 \quad (36)$$

where  $\text{Re}(\gamma_1, \gamma_2) > 0$  and  $\text{Re}(\gamma_3, \gamma_4) < 0$ . The constants  $G_1$  through  $G_7$  are known (particular solutions) and  $C, A_1, A_2, A_3, A_4$  are unknown. The constants  $\{B_i\}$  are related to  $\{A_i\}$  by

$$B_i = 16 T_c^3 L [L^2 (\gamma_i^2 - k^2) - 3]^{-1} A_i. \quad (37)$$

Moreover  $\mu$  is given by the formula

$$\mu = \frac{1}{2} M(\rho D)^{-1} + \frac{1}{2} [M^2(\rho D)^{-2} + 4(k^2 + \rho \omega / \rho D)]^{1/2}. \quad (38)$$

Thus there are six unknown constants  $(A_1, A_2, A_3, A_4, C, \delta)$ .

The jump conditions (10), when linearized, are:

$$\begin{aligned}
 [T_o] = 0, [\psi_o] = 0, [\varphi_o] = 0, [Y_o] = 0, \\
 [T_o] = -E(T_a - T_f)T_o / 2RT_o^2 \alpha L, [Y_o] = EY_f MT_o / 2RT_o^2 \rho D.
 \end{aligned}
 \tag{39}$$

These provide six relations amongst the six constants and so define a linear homogeneous system which only has a non-trivial solution for certain values of the eigenvalue  $\omega$ . In this way, stability boundaries in the wave-number, Lewis-number plane can be constructed.

Representative stability results are shown in Fig. 3 for an activation energy  $\theta$  of 15. Plotted are neutral stability boundaries for (a)  $\alpha = 1$ , (b)  $\alpha = 0.5$ , (c)  $\alpha = 0.1$ , and (d)  $\alpha = .02$ , in the Lewis number ( $Le = \lambda / \rho D C_p$ ) - scaled wave-number ( $\alpha k$ ) plane, and for various values of the Boltzman number ( $\beta$ ).

The right stability boundary (denoted by dashed curves in Fig. 3) corresponds to a pulsating or traveling wave instability ( $\text{Im } \omega \neq 0$ ), one that is not accessible for common mixtures when  $\beta = 0$ . But radiative effects push this boundary sharply to the left, to commonly achieved values of  $Le$ . Figure 4a shows the locus of the intersection of this boundary with the line  $\alpha k = c$  in the  $Le$ - $\beta$  plane for  $c = .001, 0.1, 0.5$ , and  $1.0$ , when  $\alpha = 1.0$ . The shift to the left (smaller  $Le$ ) with increasing  $\beta$  is eventually reversed. This is no different from the effect of burner-induced heat losses on the stability boundary in which displacement to the left is followed in due course by reversal of the entire curve [8].

The right stability boundary is not shown in Fig. 3c ( $\alpha = .02$ ). For small values of  $\alpha$  the roots are extremely sensitive to variations



in  $\beta$  and there are serious convergence difficulties. Sensitivity to a parameter which is only roughly defined (because of the linearized transport equation) discouraged us from examining this situation more carefully. If it is to be done it should be done in the context of a more accurate model than the one considered here.

The analogy with burner-induced losses carries over to the left stability boundary (denoted by solid curves in Fig. 3) which corresponds to the familiar cellular flames and for which  $\omega = 0$ . There is a strong tendency to suppress the instability by displacement of the curve away from  $Le = 1$ . Ronney [2] has reported that lean methane/air flames are smoothed by the addition of dust, in agreement with these results. Cellular flames in dustless mixtures are usually unsteady, a consequence of the fact that the maximum Lewis number on the stability boundary is then achieved when  $k = 0$  (see, for example, the numerical solutions of the Kuramoto-Sivashinsky equation in [9]). This characteristic does not in general survive the addition of dust. For example, when  $\alpha = 0.1$ ,  $\beta = 5$  the right most point on the boundary is located at  $\alpha k \simeq 0.42$ . Under this circumstance we anticipate a steady cellular structure on a scale defined by this special value of the wave number. The same effect is seen when burner-induced heat losses are accounted for [8], as seen in stationary polyhedral flames [10], [11].

### Concluding Remarks

We have shown that the introduction of significant radiative transport by the addition of dust to combustible mixtures can have a significant effect on the thermal-diffusive stability boundaries. The

pulsating instability can be strongly enhanced and the cellular instability suppressed. Suppression of cellular instabilities in lean methane/air mixtures has been reported by Ronney [2].

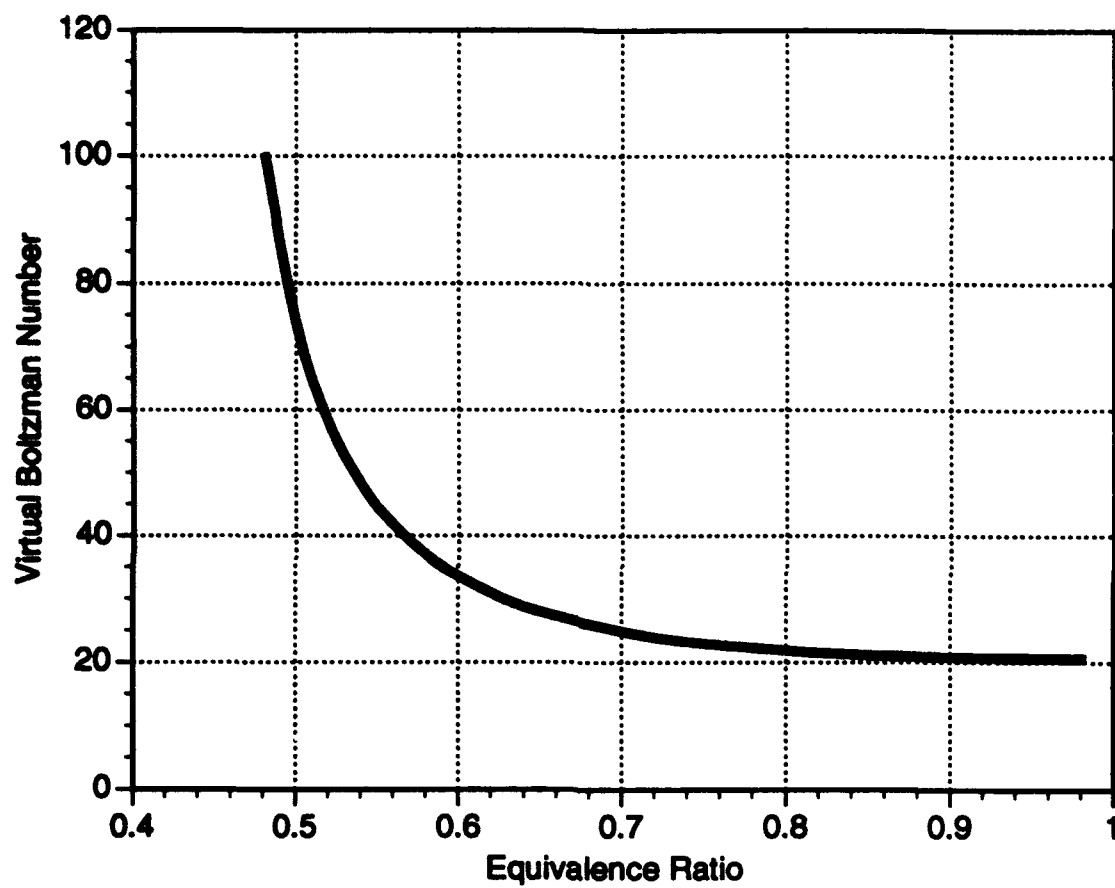
The radiation model that we have adopted is a crude one, although it appears to retain the essential physics. One of its characteristics is that it leads naturally to the definition of a Boltzman number that contains the numerical factor 16 rather than 4 (cf. Eqn. (13b)). As a consequence too much significance should not be attributed to the precise values of  $\beta$  that appear in the discussion and in the figures. The order-of-magnitude is correct however. Should more detailed experimental data become available in the future it might be appropriate to carry out a more accurate analysis, together with more detailed exploration of the stability boundary locations.

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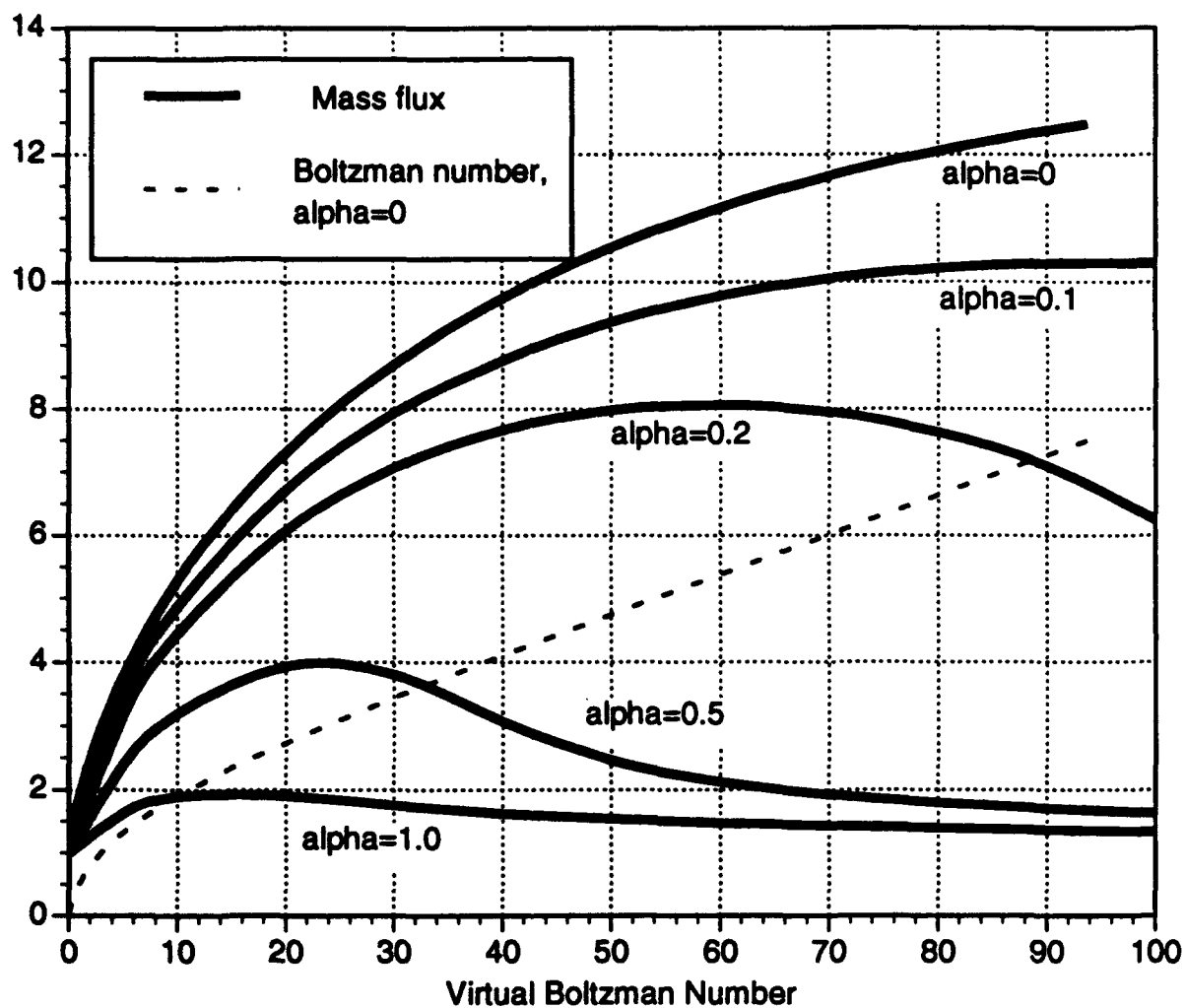
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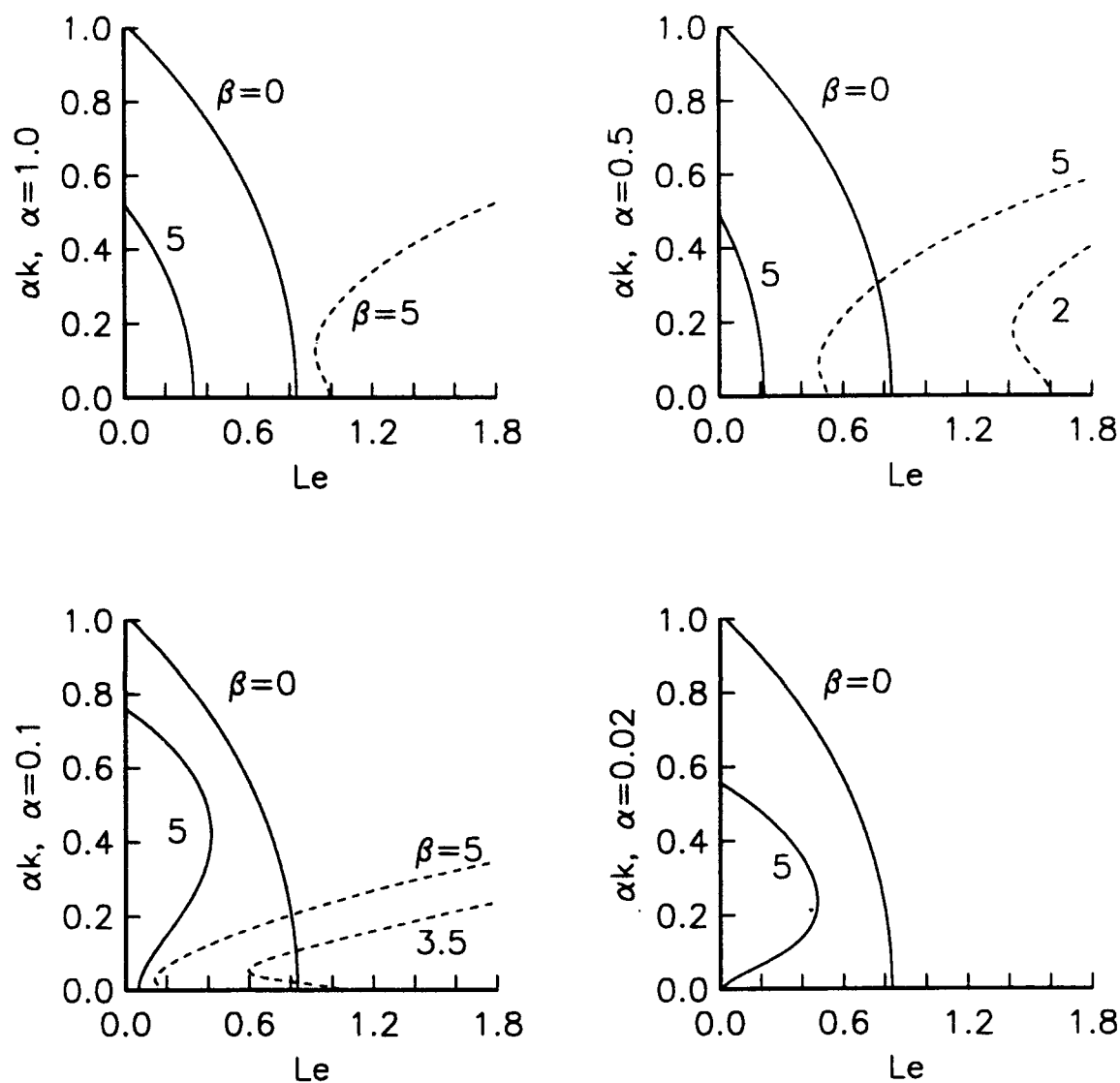
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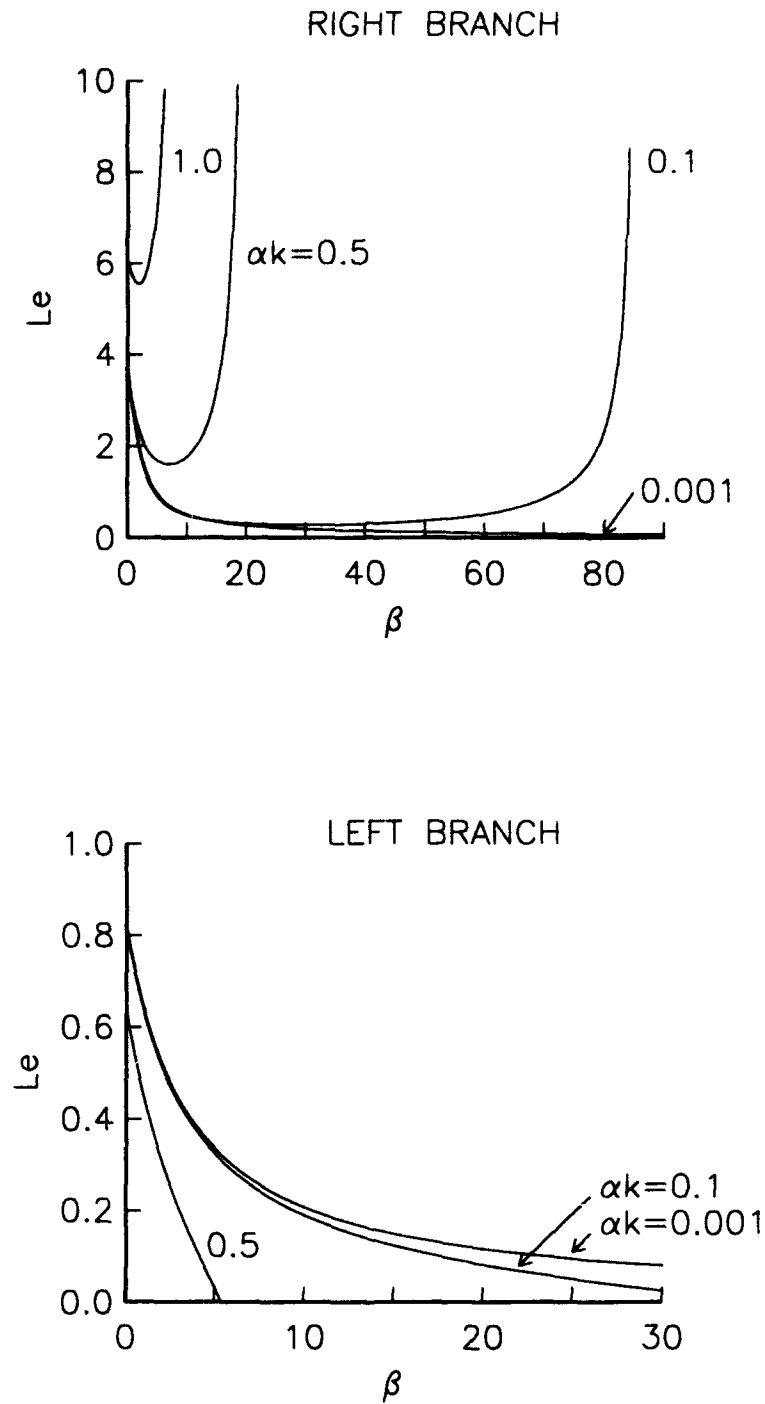
**Fig. 1** Virtual Boltzman number vs. Equivalence Ratio for lean methane/air flames, calculated using the flame temperatures and speeds obtained numerically by Giovangigli and Smooke [6].



**Fig. 2** Variations of the mass flux  $M/M_0$  with virtual Boltzmann number for  $\alpha = 0, 0.1, 0.2, 0.5$ , and  $1.0$  when  $\theta = 12$ ,  $T_a/T_f = 5$ . Also shown (dotted line) are variations of the Boltzmann number when  $\alpha = 0$ .



**Fig. 3** Stability boundaries in the wave-number - Lewis number plane for  $\theta = 10$ ,  $T_a/T_f = 5$ ,  $\alpha = 1, 0.5, 0.1, 0.02$ , and different values of  $\beta$ .



**Fig. 4** Locus of the point  $\text{Re}(\omega) = 0$ ,  $\alpha k = c$  in the Lewis number -  $\beta$  plane.

4a. Right branch,  $\theta = 15$ ,  $\alpha = 1.0$ ,  $T_a/T_f = 5$ .

4b. Left branch,  $\theta = 15$ ,  $\alpha = 1.0$ ,  $T_a/T_f = 5$ .